Kinematics & Dynamics of Machines

(4th semester, ME)

4th Semester	RME4C001	Kinematics & Dynamics of Machines	L-T-P 3-0-0	3 CREDITS

Module – I : (12 hrs)

Kinematic fundamental: Basic Kinematic concepts and definitions, Degrees of freedom, Elementary Mechanism : Link, joint, Kinematic Pair, Classification of kinematic pairs, Kinematic chain and mechanism, Gru ebler's criterion, Inversion of mechanism, Grashof criteria, Four bar linkage and their inversions, Single slider crank mechanism, Double slider crank mechanism and their inversion. Transmission angle and toggle position, Mechanical advantage.

Kinematic Analysis : Graphical analysis of position, velocity and acceleration of four bar and Slider crank mechanisms. Instantaneous centre method, Aronhold-Kennedy Theorem, Rubbing velocity at a Pin-joint.Coriolis component of acceleration.

Module – II : (10 hrs)

Gear and Gear Trains: Gear Terminology and definitions, Theory of shape and action of tooth properties and methods of generation of standard tooth profiles, Standard proportions, Force analysis, Interference and Undercutting, Methods for eliminating Interference, Minimum number of teeth to avoid interference. Analysis of mechanism Trains: Simple Train, Compound train, Reverted train, Epicyclic train and their applications.

.

Module – III : (8 *hrs*)

Combined Static and Inertia Force Analysis: Inertia forces analysis**,** velocity and acceleration of slider crank mechanism by analytical method, engine force analysis - piston effort, force acting along the connecting rod, crank effort. dynamically equivalent system, compound pendulum, correction couple.

Module – IV : (8 *hrs*)

Friction Effects: Screw jack, friction between pivot and collars, single, multi-plate and cone clutches, anti friction bearing, film friction, friction circle, friction axis.

Flexible Mechanical Elements: Belt, rope and chain drives, initial tension, effect of centrifugal tension on power transmission, maximum power transmission capacity, belt creep and slip.

Module – V : (7 *hrs*)

Brakes & Dynamometers : Classification of brakes, Analysis of simple block, Band and internal expanding shoe brake, Braking of a vehicle. Absorption and transmission dynamometers, Prony brake, Rope brake dynamometer, belt transmission, epicyclic train, torsion dynamometer.

Books:

- Kinematics and Dynamics of Machinery by R L Norton, Tata MacGraw Hill
- Theory of Machines and Mechanisms by John J. Uicker Jr., Gordon R. Pennock and Joseph E. Shigley, Oxford University Press
- Theory of Machines by S.S.Rattan, Tata MacGraw Hill
- Theory of Machines by Thomas Bevan, CBS Publications
- Kinematics and Dynamics of Machinery by Charles E. Wilson and J.Peter Saddler,

Chapter-1

Module-1

Kinematics Fundamental

Defination:

Kinematics and dynamics of machine or theory of machine may be defined as the branch of engineering science which deals with the study of relative motion between various parts of a machine and the forces which acts on them.

Sub division of theory of machine:

(1) **Kinematics:**

It is the branch of theory of machine which deals with the relative motion between various parts of the machine without considering the cause force.

(2) **Kinetics:**

It is the branch of theory of machine which deals with the relative motion between various parts of the machine by considering inertia force into account.

(3) **Dynamics:**

It is the branch of theory of machines which deals with the force and their effects while acting upon the machine parts in motion.

(4) **Statics:**

It is that branch of theory of machine which deals with the forces and their effects while the machine parts at rest.

(5) **Machine:**

- Machine is a device which receives energy in some available form and utilies it to do some perticular type of work.
- Machine consists of a number of parts or bodies.

Kinematic chain :

when the kinematic pairs are coupled in such a way that the last link is joint to the first link to transmit definite motion (i.e completely or successfully constant motion) it is called a kinematic chain.

In other words, A kinematic chain may be defined as a combination of kinematic pairs join in such a way that each link forms a part of two pairs and the relative motion between the links or elements is completely or successfully constant.

If each link is assume to form two pairs with two adjecent link then the relative motion between the no. of pairs (p) forming a kinematic chain and the no. of links (l) is may be expressed in the form of equation

$$\boxed{l = 2p - 4} \quad \text{---} \quad (i)$$

The relationship between no. of links (l) and no. of joints (j) which constitute a kinematic chain is given by the expression

$$\boxed{j = (3/2)\, l - 2} \quad \text{---} \quad (ii)$$

Note:

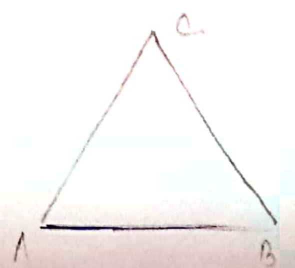
The equations (i), (ii) are applicable to kinematics chains in which lower pair are used. These equations may also be apply to kinematics chain in which higher pairs are used. In that case each higher pair may be taken as equivalent to two lower pairs with an aditional element on link.

Ex: (i)

Link (l) = 3

joints (j) = 3

pairs (p) = 3



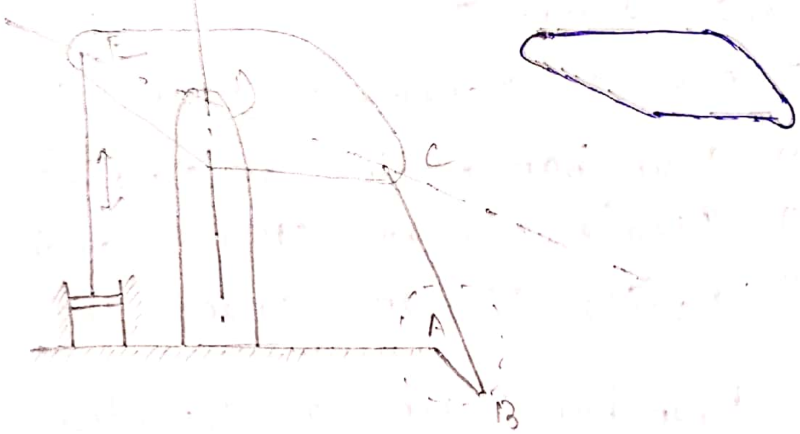
It consists of 4 links, each of them forms a turning pair at A, B, C & D. The four links may be of different length.

According to Grashof's law for a four bar mechanism, the sum of the shortest and longest link length should not be greater than the sum of the remaining two lengths if there is to be continous relative motion between two links.

A very important consideration in designing a mechanism is to ensure that the input crank makes a complete revolution relative to the other links. The mechanism in which no link makes a complete revolution will not be useful. In a four bar chain, one of the link in perticular the shortest link will make a complete revolution to the other three links if it satisfy Grashof's law, such a link is known as crank.

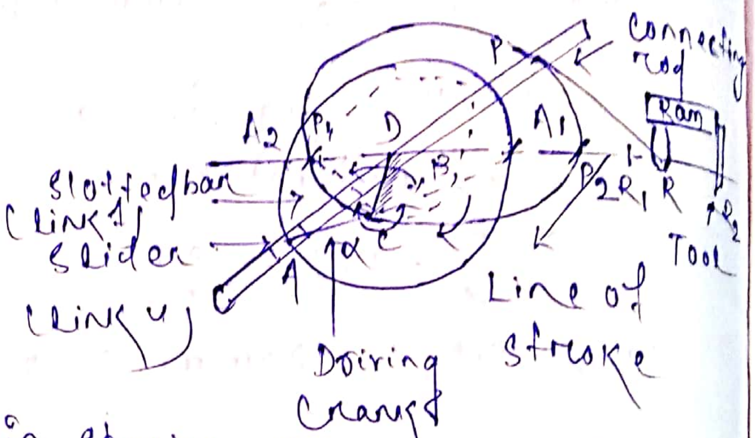
Inversion of Four bar chain :

(i) Beam engine (crank & lever mechanism) :-



In this mechanism when the crank rotates about the fixed centre 'A', the lever oscillates about a fix centre 'D'.
The end 'E' of the lever is connected to a piston rod which reciprocates due to the rotation of the crank. In other words the purpose of this mechanism is to convert rotary motion into reciprocating motion.

Whitworth Quick Return motion Mechanism :-



This mechanism is mostly used in shaping and slotting machines. In this mechanism the link CD (link 2) forming the turning pair is fixed. The driving crank CA (link 3) rotates at a uniform angular speed. The slider (link 4) attach to the crank pin at A. Slides along the slotted bar PA (link 1) which oscillates at a pivoted point D. The connecting rod PR carries the ram at R to which a cutting tool is fixed. When the driving crank CA moves from position CA_1 to CA_2 (link DB from position DP_1 to DP_2 through an angle α in clockwise direction then the tool moves from the position R_1 to R_2.

$$P_1P_2 = R_1R_2$$

$$\Rightarrow P_1D + DP_2 = R_1R_2$$

$$\Rightarrow 2P_1D = R_1R_2$$

$$\Rightarrow 2DP = R_1R_2 = \text{Length of stroke}$$

A little consideration will show that the time taken during the left hand to right hand movement of the ram will be equal to the time taken by the driving crank to move from CA_1 to CA_2. Similarly the time taken during the right to left movement of the ram will be equal to the time taken by the driving crank to move from CA_2 to CA_1.

Important Points

From the above problem, it is clear that

(*i*) If V_A is known in magnitude and direction and V_B in direction only, then velocity of point B may be determined in magnitude.

(*ii*) The velocity of any other point C lying on the link AB can be determined in magnitude and direction.

(*iii*) The magnitude of velocities of the points on a rigid link is inversely proportional to the distances of the points from the instantaneous centre and is $\perp$ to the line joining the points to the instantaneous centre.

3.3. Analysis of Reciprocating Engine Mechanism By Instantaneous Centre Method

Fig. 3.8 shows a reciprocating engine mechanism in which AB is the connecting rod and BC is the crank. The crank BC is rotating at a uniform angular velocity in the clockwise direction about point C. The point A connected to the piston rod and connecting rod is having to and fro motion in the horizontal plane. The connecting rod is having a combined motion of translation and rotation.

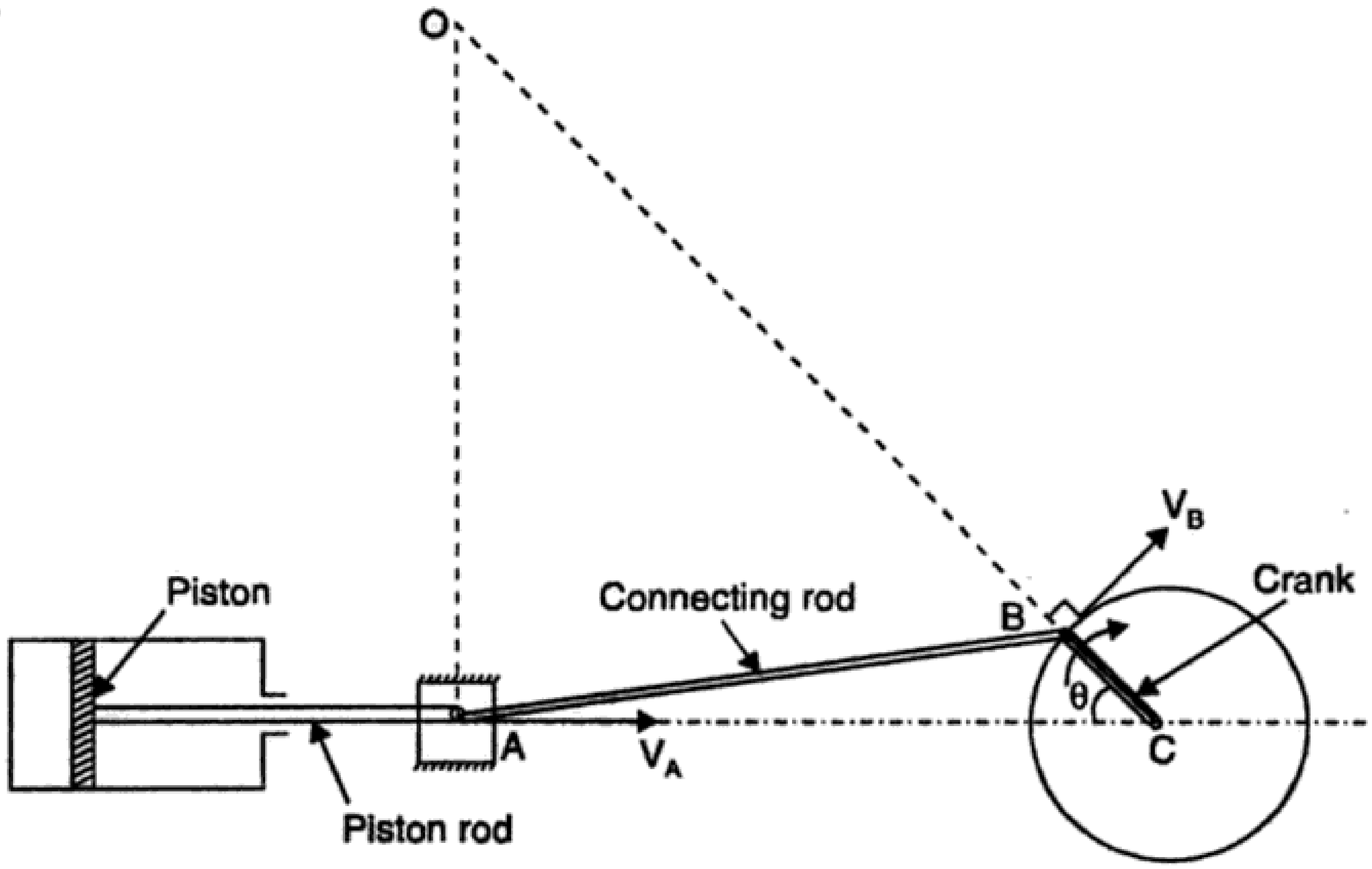


Fig. 3.8

Let N = Revolution of crank in r.p.m.

ω = Angular velocity of crank

r = Radius of crank, *i.e.*, length BC

L = Length of connecting rod AB.

Then
$$\omega = \frac{2\pi N}{60}.$$

Linear velocity of point $B = V_B = \omega \times BC = \omega \times r$ ...(*i*)

The direction of the linear velocity at B is along the tangent at B to the crank circle. Hence V_B will be acting at right angles to BC as shown in Fig. 3.8. The velocity at A (*i.e.*, V_A) is in the horizontal direction acting along AC for the given position of the crank. Since the directions of V_A and V_B are known, the position of the instantaneous centre can be determined by drawing perpendiculars to the directions of the velocities at A and B. The two perpendiculars meet at O, which is the instantaneous centre for the connecting rod AB. Thus for the vector ac horizontally (*i.e.*, parallel to the path of motion of C which is along CA) to represent the velocity C with respect to A or simply the velocity of C *i.e.*, v_{CA} or v_C. The vectors bc and ac intersect at c. Then abc represents the velocity diagram for the given slider crank mechanism.

By measurement, we find velocity of C with respect to B,

$$v_{CB} = \text{Vector } bc = 340 \text{ cm/s}$$

and velocity of C with respect to A or simply velocity of C,

$$v_{CA} \text{ or } v_C = \text{Vector } ac = 400 \text{ cm/s.}$$

(*i*) ∴ Velocity of slider $C = v_C =$ **400 cm/s. Ans.**

(*ii*) Angular velocity of connecting rod CB is given by,

$$\omega_{CB} = \frac{v_{CB}}{\text{Length } BC} = \frac{340}{60} = \mathbf{5.67 \text{ rad/s.}} \quad \textbf{Ans.}$$

(*iii*) *Linear velocity of the mid-point of the connecting rod.*

The mid-point of the connecting rod CB is point D in space diagram. Hence the corresponding point d in the velocity diagram will be the mid-point of vector bc. Now join ad. Then vector ad represents the velocity of the mid-point D of the connecting rod *i.e.*, v_D.

By measurement, we find that,

$$v_D = \text{Vector } ad = \mathbf{410 \text{ cm/s.}} \quad \textbf{Ans.}$$

3.12. Rubbing Velocity at a Pin-Joint

The two links 1 and 2 are connected by means of a pin-joint as shown in Fig. 3.32.

Let ω_1 = Angular velocity of link 1,

ω_2 = Angular velocity of link 2, and

r = Radius of the pin at the joint.

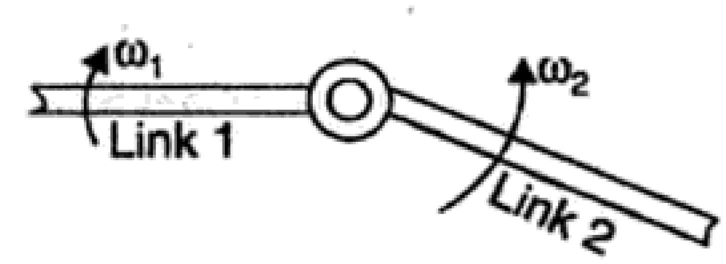


Fig. 3.32

The rubbing velocity is defined as the *algebraic difference* between the angular velocities of the two links which are connected by pin-joints, multiplied by the radius of the pin.

Hence the rubbing velocity at the pin joint, when the two connected links move in opposite direction is given by,

Rubbing velocity $= r(\omega_1 + \omega_2)$ ...(3.7)

But if the two connected links move in the same direction, then

Rubbing velocity $= r(\omega_1 - \omega_2)$ ...(3.8)

If a pin connects one sliding member and the other turning member (For example gudgeon pin of a connecting rod) the angular velocity of the sliding member is zero and hence the velocity of rubbing will be given by,

Rubbing velocity $= r \,.\, \omega$ ...(3.9)

where r = Radius of the pin and

ω = Angular velocity of the turning member.

Problem 3.10. *The oscillating link OAB of a mechanism shown in Fig. 3.33, is pivoted at O and is moving at 90 r.p.m. anti-clockwise. If OA = 15 cm, AB = 7.5 cm and AC = 25 cm, then calculate :*

(*i*) *the velocity of the block C,*

(*ii*) *angular velocity of the link AC and*

(*iii*) *the rubbing velocities of the pins at O, A and C assuming that these pins are of equal diameter of 2 cm. The oscillating link OAB makes an angle of 15 with the vertical as shown in Fig. 3.33(a).*

(b) Internal gearing :-

Internal gearing, the gears of the two shafts mesh internally with each other. The larger of these two wheels is called annular wheel and the smaller wheel is called pinion.

→ The motion of the two wheels is always like.

(c) Rack and pinion :-

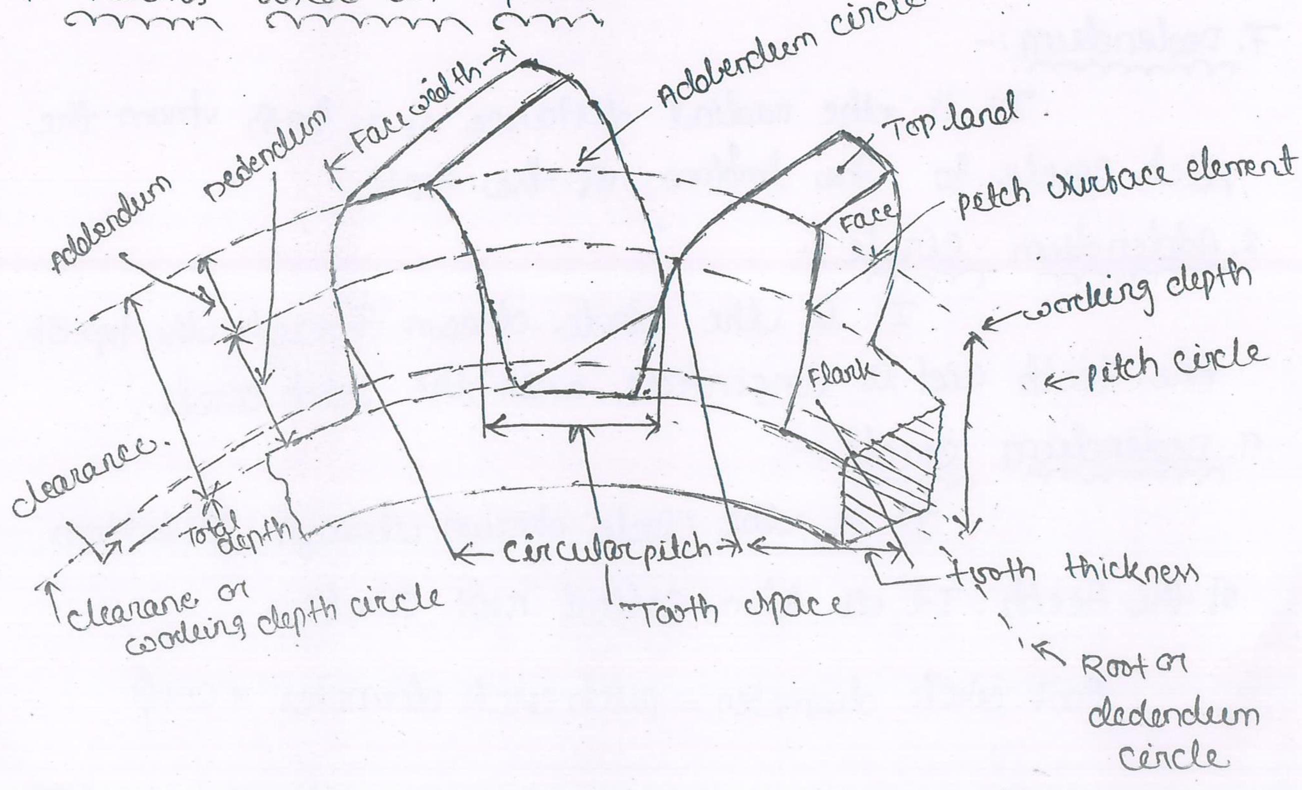
The gear of a shaft meshes externally and internally with the gears in a straight line this type of gear is called Rack and pinion. The straight line gear is called rack and the circular wheel is called pinion.

→ Here convert linear motion into rotary motion and vice - versa.

4. According to position of teeth on the gear surface :-

ⓐ straight ⓑ inclined ⓒ curved.

* Terms used in Gears :-



→ The epicyclic gear train may be simple or compound type.

Velocity ratio of epicyclic gear train :

Two methods are there to find out the velocity ratio of epicyclic gear train.

(1) Algebraic method
(2) Tabular method

(1) Algebraic Method :

Here the motion of each element of epicyclic gear train relative to the arm is written in form of an equation.

Let N_A = Speed of gear wheel 'A'
N_B = " " 'B'
N_C = " of arm 'C'
T_A = No. of teeths on wheel 'A'
T_B = " " wheel 'B'

Since the gear 'A' and 'B' are messing directly therefore they will revolve in opposite direction.

$$\frac{\text{Speed of wheel 'B' w.r.t arm 'C'}}{\text{Speed of wheel 'A' w.r.t arm 'C'}} = -\frac{T_A}{T_B}$$

$$\Rightarrow \frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B} \quad \text{--- (1)}$$

$$\Rightarrow \frac{N_B - N_C}{-N_C} = -\frac{T_A}{T_B} \qquad (\because \text{wheel 'A' is fixed})$$

$$\Rightarrow -\frac{N_B}{N_C} + 1 = -\frac{T_A}{T_B}$$

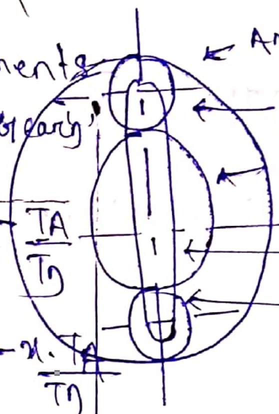
$$\Rightarrow \frac{N_B}{N_C} = 1 + \frac{T_A}{T_B} \Rightarrow \boxed{N_B = N_C\left(1 + \frac{T_A}{T_B}\right)}$$

Q. If 2 planet gears 'B' and 'C' having 30 teeth each are attached to arm 'E' as shown in fig. and gear 'A' is having 40 teeth then find the no. of revolutions made by the arm when

(i) Gear 'A' makes 1 revolution clockwise and 'B' makes half revolution anticlockwise

(ii) Gear 'A' makes 1 revolution clockwise and 'D' is stationary

Sl. No.	Operations	Arm 'E'	Gear 'A'	Gear 'B' or Gear 'C'	Gear 'D'
1.	Arm is fixed. Give +1 rev. to gear 'A'	0	+1	$-\frac{T_A}{T_B}$	$-\frac{T_A}{T_D}$
2.	Arm is fixed. Give +x rev. to gear 'A'	0	+x	$-x\frac{T_A}{T_B}$	$-x\cdot\frac{T_A}{T_D}$
3.	Add +y to all the elements	+y	+y	+y	+y
4.	Resultant motion	y	x+y	$y-x\frac{T_A}{T_B}$	$y-x\cdot\frac{T_A}{T_D}$



$$\frac{N_B}{N_A} = -\frac{T_A}{T_B}$$

$$\Rightarrow N_B = -N_A\cdot\frac{T_A}{T_B} = N_C$$

$$\frac{N_D}{N_B} = \frac{T_B}{T_D}$$

$$\Rightarrow T_D = N_B\cdot\frac{T_B}{T_D}$$

$$= -N_A\cdot\frac{T_A}{\cancel{T_B}}\times\frac{\cancel{T_B}}{T_D}$$

$$= -N_A\cdot\frac{T_A}{T_D}$$

(i) $N_A = -1$ $\quad\therefore x+y = N_A = -1 \quad ---(i)$

$N_D = 1/2$ $\quad\therefore N_D = y - x\cdot\frac{T_A}{T_D} = 1/2$

$N_E = ?$ $\quad = y - x\cdot\frac{40}{100} = \frac{1}{2} \quad ---(ii)$

6.4. Laws of Dry Friction

The friction, that exists between two surfaces which are not lubricated, is known as dry friction. The two surfaces may be at rest or one of the surface is moving and other surface is at rest. The following are the laws of solid friction :

1. The force of friction acts in the opposite direction in which surface is having tendency to move.
2. The force of friction is equal to the force applied to the surface, so long as the surface is at rest.
3. When the surface is on the point of motion, the force of friction is maximum and this maximum frictional force is called the limiting friction force.
4. The limiting frictional force bears a constant ratio to the normal reaction between two surfaces.
5. The limiting frictional force does not depend upon the shape and areas of the surfaces in contact.
6. The ratio between limiting friction and normal reaction is slightly less when the two surfaces are in motion.
7. The force of friction is independent of the velocity of sliding of one surface over the other.

The above laws of solid friction are also called laws of static and dynamic friction. The last law is valid only within the limits. It has been found that the friction force decreases slightly with the increase in velocity.

Problem 6.1. *A body of weight 100 Newtons is placed on a rough horizontal plane. Determine the co-efficient of friction if a horizontal force of 60 Newtons just causes the body to slide over the horizontal plane.*

Sol. Given :

Weight of body, $W = 100$ N

Horizontal force applied, $P = 60$ N

$\therefore$ Limiting force of friction, $F = P = 60$ N

Let μ = co-efficient of friction.

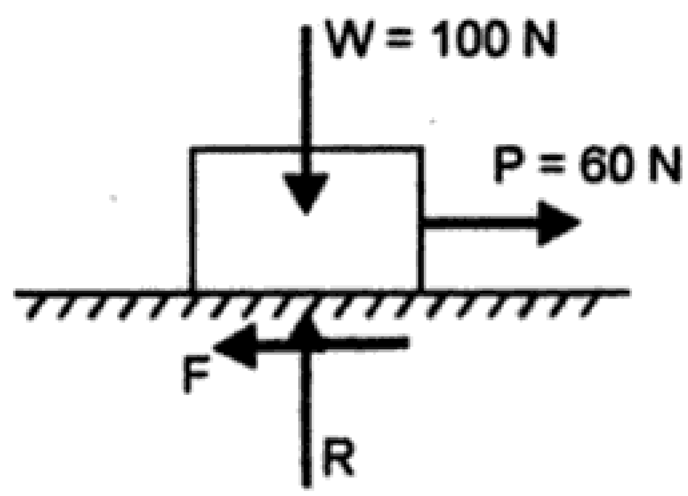


Fig. 6.5

The normal reaction of the body is given as

$$R = W = 100 \text{ N}$$

Using equation (6.1),

$$F = \mu R$$

or $$\mu = \frac{F}{R} = \frac{60}{100} = \mathbf{0.6.} \quad \textbf{Ans.}$$

Problem 6.2. *A body of weight 200 N is placed on a rough horizontal plane. If the co-efficient of friction between the body and the horizontal plane is 0.3, determine the horizontal force required to just slide the body on the plane.*

Sol. Given :

Weight of body, $W = 200$ N

Co-efficient of friction, $\mu = 0.3$

Normal reaction, $R = W = 200$ N

Let F = horizontal force which causes the body to just slide over the plane.

Using equation (6.1), $F = \mu R = 0.3 \times 200 = \mathbf{60\ N.}$ **Ans.**

Problem 6.3. *The force required to pull a body of weight 50 N on a rough horizontal plane is 15 N. Determine the co-efficient of friction if the force is applied at an angle of 15° with the horizontal.*

Sol. Given :

Weight of the body, $W = 50$ N

Resolving forces normal to the plane,

$$R = 500 \cos 30° = 500 \times .866 = 433 \text{ N}$$

Substituting the value of R in equation (*i*), we get

$$500 \sin 30° + \mu \times 433 = 350$$

or $$500 \times 0.5 + 433\,\mu = 350$$

or $$433\,\mu = 350 - 500 \times 0.5 = 350 - 250 = 100$$

$\therefore$ $$\mu = \frac{100}{433} = \mathbf{0.23}. \quad \textbf{Ans.}$$

Problem 6.10. *A body of weight 450 N is pulled up along an inclined plane having inclination 30° to the horizontal at a steady speed. Find the force required if the co-efficient of friction between the body and the plane is 0.25 and force is applied parallel to the inclined plane. If the distance travelled by the body is 10 m along the plane, find the work done on the body.*

Sol. Given :

Weight of body, $W = 450$ N

Inclination of plane, $\alpha = 30°$

Co-efficient of friction, $\mu = 10.25$

Distance travelled by body = 10 m

Let the force required $= P$.

The body is in equilibrium under the action of forces shown in Fig. 6.17.

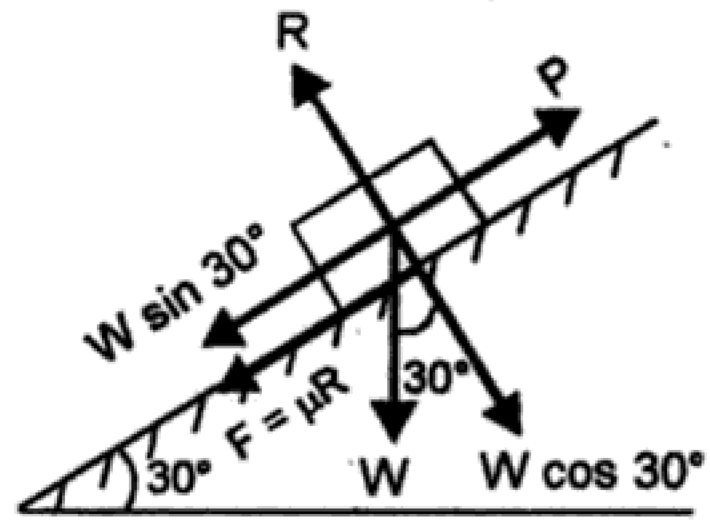


Fig. 6.17. Body moving up.

Resolving forces along the plane,

$$P = W \sin 30° + \mu R = 450 \times 0.5 + 0.25 \times R$$

or $$P = 225 + 0.25R \quad \text{...(}i\text{)}$$

Resolving forces normal to the plane,

$$R = W \cos 30° = 450 \times 0.866 = 389.7 \text{ N}$$

Substituting the value of R in equation (*i*),

$$P = 225 + 0.25 \times 389.7 = \mathbf{322.425\ N}. \quad \textbf{Ans.}$$

Work done on the body = Force × Distance travelled in the direction of force

= 322.525 × 10 N m = 3224.25 N m

= **3224.25 J** (where J = Joules = N m). **Ans.**

Problem 6.11. *An effort of 200 N is required just to move a certain body up an inclined plane of angle 15°, the force acting parallel to the plane. If the angle of inclination of the plane is made 20°, the effort, required, again applied parallel to the plane, is found to be 230 N. Find the weight of the body and the co-efficient of friction.* (AMIE Summer, 1980)

Sol. Given :

Effort required, $P_1 = 200$ N ; when inclination, $\theta_1 = 15°$

Effort required, $P_2 = 230$ N ; when inclination, $\theta_2 = 20°$.

In both the cases, the effort is applied parallel to the inclined plane and body is just to move up. Hence the force of friction ($F = \mu R$) will be acting downwards.

1st Case

$$P_1 = 200 \text{ N}, \theta_1 = 15°$$

$$= 2 \times \left[\frac{0.25 \times 1005.31 \times (0.09 + 0.05)}{2} \right] \quad [\because \text{ From } (ii), W = 1005.31]$$

$$= 35.186 \text{ Nm.}$$

Now let us find the angular acceleration, when total torque is 35.186 Nm.

But Torque = M.O.I. $\times \alpha$ where α is angular acceleration.

or $$T = I \times \alpha$$

$$\therefore \quad 35.186 = 5.5 \times \alpha \quad (\because \; T = T^* = 35.186, \text{ and } I = 5.5 \text{ kg m}^2)$$

$$\therefore \quad \alpha = \frac{35.186}{5.5} = 6.397 \text{ rad/s}^2$$

The machine starts from rest. After some time the final angular speed of the machine will be corresponding to speed of shaft (*i.e.* corresponding to 240 r.p.m.).

$\therefore$ Final angular speed of machine, $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 8\pi$ rad/s

Initial angular speed of machine = $\omega_0 = 0$

Angular acceleration, $\alpha = 6.397$ rad/s^2

Let t = time required for the machine to attain the full speed.

Using $\omega = \omega_0 + \alpha t$, we get $8\pi = 0 + 6.397 \times t$

$$\therefore \quad t = \frac{8\pi}{6.397} = \mathbf{3.928 \text{ s.}} \quad \textbf{Ans.}$$

Hence the full speed will be attained by the machine in 3.928 seconds.

(*ii*) *Energy lost in slipping of the clutch*

The driving shaft is rotating at a uniform speed of 240 r.p.m. *i.e.* at a uniform angular speed of $\frac{2\pi \times 240}{60} = 8\pi$ rad/s, whereas the machine starts from rest and attains the full speed *i.e.* 8π rad/s in time 3.928 seconds. Let us find the angles turned by driving shaft and the machine in time 3.928 seconds.

Angle turned by the driving shaft,

$$\theta_1 = \omega \times t \quad (\because \text{ shaft is rotating with a uniform angular velocity})$$

$$= 8\pi \times 3.928 \text{ rad} = 98.72 \text{ rad.}$$

The angle turned by the machine (or driven shaft) will be obtained by using $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $\left(\text{corresponding to } s = ut + \frac{1}{2}at^2\right)$.

Let θ_2 = Angle turned by the driven shaft

$$\therefore \quad \theta_2 = 0 \times 3.928 + \frac{1}{2} \times 6.397 \times (3.928)^2 \quad (\because \; \omega_0 = 0, \alpha = 3.928, t = 3.928)$$

$$= 0 + 49.35 = 49.35 \text{ rad.}$$

Energy lost in friction due to clutch slip

$$= \text{Frictional torque} \times \text{Angle of slip}$$

$$= 35.186\,(\theta_1 - \theta_2) = 35.186\,(98.72 - 49.35) = \mathbf{1737.13 \text{ Nm.}} \quad \textbf{Ans.}$$

Problem 6.29. *For the problem 6.28, determine the ratio of power transmitted with uniform wear to that with uniform pressure. Also find the intensity of pressure if the condition of uniform pressure is considered and the same axial load is developed as in case of uniform wear.*

$R = T \sin\frac{\delta\theta}{2} + (T+\delta T)\sin\delta\theta/2$

Since the angle $\delta\theta$ is very small, $\sin\frac{\delta\theta}{2}$ can be written as $\frac{\delta\theta}{2}$

Hence the above equation becomes as

$$R = T \times \delta\theta/2 + (T+\delta T) \times \delta\theta/2$$

$$= T \times \delta\theta/2 + T\frac{\delta\theta}{2} + \delta T \times \frac{\delta\theta}{2}$$

$$= T \times \delta\theta + \frac{\delta T\,\delta\theta}{2}$$

$$= T \times \delta\theta \text{ —(i)} \quad \left(\because \text{ Neglecting the small quantity } \frac{\delta\theta \times \delta T}{2} \right)$$

Now resolving all the forces vertically, we get

$$F = (T+\delta T)\cos\delta\theta/2 - T\cos\delta\theta/2$$

Since $\delta\theta$ is very small, hence $\cos\frac{\delta\theta}{2}$ reduces to unity i.e 1.

Hence the above equation becomes as

$$F = (T+\delta T) - T = \delta T$$

$$\mu R = \delta T \qquad \therefore (F = \mu R)$$

$$R = \frac{\delta T}{\mu} \text{ ——— (ii)}$$

Equating the two values of R given by eqn (i) & (ii). we get

$$T \times \delta\theta = \frac{\delta T}{\mu}$$

$$\frac{\delta T}{T} = \mu \cdot \delta\theta$$

Intigrating the above equation between the limits T_2 & T_1

we get $\int_{T_1}^{T_2} \frac{dT}{T} = \int \mu \cdot d\theta = \mu \int d\theta$

$$\log_e T_1/T_2 = \mu \times \theta$$

$$\boxed{\frac{T_1}{T_2} = e^{\mu \cdot \theta}}$$

Angle of contact for open belt drive :- With an open belt drive, the belt will begin to slip on the smaller pulley, since the angle of lap is smaller on this pulley than on the larger pulley. The angle θ should be taken as the minimum angle of contact. The angle of contact of lap (θ) at the smaller

$$\therefore \quad P = \frac{T_2 \times 130}{390}$$

$$= \frac{277.77 \times 130}{390} = \mathbf{92.59\ N.} \quad \textbf{Ans.}$$

(ii) Number of turns of the flywheel before it comes to rest.

Let n = Number of turns of the flywheel before it comes to rest.

The kinetic energy of the rotation of the flywheel is used to overcome the workdone due to braking torque (T_B), before the flywheel comes to rest.

Now K.E. of the rotation* of flywheel

$$= \frac{1}{2} \times I + \omega^2$$

$$= \frac{1}{2} \times mk^2 \times \omega^2 \qquad (\because \; I = mk^2)$$

$$= \frac{1}{2} \times mk^2 \times \left(\frac{2\pi N}{60}\right)^2 \qquad \left(\because \; \omega = \frac{2\pi N}{60}\right)$$

$$= \frac{1}{2} \times 300 \times 0.35^2 \times \left(\frac{2\pi \times 200}{60}\right)^2$$

$$= 8060.17 \text{ Nm} \qquad ...(iii)$$

$\therefore$ Work done by the braking torque in 'n' number of turns of the flywheel

$$= T_B \times \text{Angular displacement in } n \text{ turns}$$

$$= T_B \times 2\pi \times n \qquad (\because \; \text{Angular displacement for one turn} = 2\pi)$$

$$= 39 \times 2\pi \times n \qquad ...(iv)$$

But K.E. of flywheel = Work done by braking torque

$$\therefore \quad 8060.17 = 39 \times 2\pi \times n$$

$$\therefore \quad n = \frac{8060.17}{39 \times 2\pi} = \mathbf{32.89.} \quad \textbf{Ans.}$$

(iii) Time taken by flywheel to come to rest after applying the brake

$$N = 200 \text{ r.p.m.}$$

This means that 200 revolutions are made in one minute. The flywheel comes to rest after applying the brake in 32.89 revolution. Let us find the time for 32.89 revolution.

Time for 200 revolution $= 1$ minute

Time for 1 revolution $= \frac{1}{200}$ min

Time for 32.89 revolution $= \frac{1}{200} \times 32.89 = 0.16445$ min

$= 0.16445 \times 60$ seconds

$=$ **9.867 seconds. Ans.**

Problem 8.8. *Fig. 8.12 shows a simple band brake which is applied on a drum of diameter 400 mm. The drum is rotating at 180 r.p.m. The angle of lap of the band on the drum is 270° and co-efficient of friction is 0.25. One end of the band is attached to a fixed pin (i.e. fulcrum) and other end to the lever arm at a distance*

---

*K.E. due to linear velocity $= \frac{1}{2} mV^2$ whereas the K.E. due to rotation $= \frac{1}{2} I \times \omega^2$ where $I = mk^2$.